PART 3

3

BUSINESS INVESTMENT DECISIONS— CAPITAL BUDGETING

CAPITAL BUDGETING

CHAPTER OUTLINE

The money companies spend in the normal course of business can be divided into two categories. Funds are expended on an everyday basis to buy inventory, pay expenses, and compensate employees. These expenditures can be thought of as short term in that they support daily activity.

In addition to such short-term expenditures, firms spend large sums on special projects from time to time. For example, machines normally wear out and need to be replaced every few years. The replacement expenditures tend to be relatively large, but they are infrequent. New business ventures provide a second example because they generally require initial spending to get started. Start-up amounts are usually large, but the opportunities don't come along often. Spending on things like these is long term because the projects involved tend to last for long periods. As a general rule, money spent on long-term projects is called *capital*.

The field known as **capital budgeting** *involves planning and justifying how capital dollars are spent on long-term projects.* It provides methods through which projects are evaluated to decide whether they make sense for a particular business at a point in time. It also provides a basis for choosing between projects when more than one is under consideration at the same time.

CHARACTERISTICS OF BUSINESS PROJECTS

PROJECT TYPES AND RISK

Projects fit into three general categories: replacement, expansion, and new venture. We've already used the first and the last of these as examples in the opening section. Expansion simply involves

doing something the firm already does on a larger scale. It usually requires investing money in additional resources and equipment similar to those already on hand.

A risk is associated with investing money in any project. For now we'll define that risk simply as the chance of making less on the project than management expects when the decision to go ahead is made.

Broadly speaking, risk varies with project type, increasing as we move from replacement to expansion to new venture. A replacement is the safest endeavor because we're doing something that's already being done. Expansion projects are riskier because they're based on a forecast increase in demand for the company's product that may not materialize. Finally, the riskiest project is a new venture, something the company hasn't done before. No one ever knows whether they'll be successful at something they haven't tried.

STAND-ALONE AND MUTUALLY EXCLUSIVE PROJECTS

Projects can be evaluated in either of two contexts. The first involves a proposal without a competing alternative. For example, suppose an old machine is wearing out and there's only one replacement product on the market. The choice is simply whether to buy the new machine (the project) or to do nothing.

This single-project situation is known as a **stand-alone project**. We need to decide on a project's viability by itself, standing alone. Is it a good deal for the company or not? Another way to think of it is to say that no other project is currently competing for the resources required to do this one.

The second situation involves choosing between projects. It occurs either when there's more than one way to do something, or two different things are proposed but there's only enough money to do one of them. As an example of the first circumstance, assume the worn-out machine can be replaced by either of two new ones. Suppose the first option is relatively cheap to buy but is expensive to operate and produces low-quality output. The second costs more initially but runs less expensively and produces better product. Which should the firm choose? Notice that it must choose one or the other, because only one replacement is required. In this context, the alternatives are said to be **mutually exclusive**, since choosing either excludes the other.

Sometimes projects can be mutually exclusive even if they're totally different physically. That occurs when a firm has only enough resources to do one project at a time. For example, suppose an electronics firm has new venture opportunities in computer technology and radio transmission, but it has only enough money to undertake one new idea. The projects are mutually exclusive because doing one precludes doing the other, even though from business and technical viewpoints they're entirely separate. Further, the limiting resource doesn't have to be money. It might be trained personnel, plant capacity, or management's time.

PROJECT CASH FLOWS

The first step in the capital budgeting process requires that any project under consideration be represented as a series of cash flows that is incremental to the business.

This requirement is easiest to picture in the context of a new venture. Imagine that such a new business will take an initial investment of $50,000, will lose $10,000 in the first year, and is expected to generate $15,000 in cash each year for the next five years before being shut down. For capital budgeting purposes, the project can be summarized as just that series of yearly cash outflows and inflows. If we call the cash

flow in the ith year C_i, and let C_0 be the initial investment at the beginning of year 1, we can represent the project as follows, where the numbers in parentheses are negative and represent outflows.

C_0	$(50,000)
C_1	(10,000)
C_2	15,000
C_3	15,000
C_4	15,000
C_5	15,000
C_6	15,000

Business projects involve **early cash outflows** and later inflows. The **initial outlay** is required to get started.

It's important to notice the pattern of cash flows shown here. Projects nearly always involve an initial outflow of funds followed by inflows at later dates. C_0 is virtually always negative because it represents the *initial outlay* necessary to get a project started. The remaining figures tend to be positive, although they may include some negatives as in the example.

It's conceptually easy to identify incremental cash flows for a new venture. The representation can be more difficult to see when we're talking about a replacement project. Then incremental cash flows are things like savings on fuel and maintenance or improved profitability due to higher quality product. That kind of incremental cash flow can be hard to quantify.

In fact, the most difficult and inaccurate part of capital budgeting is estimating project cash flows. For the time being we'll proceed by assuming the estimates are given for the projects we'll be talking about. In the next chapter, we'll return to the issue and consider cash flow estimation in more detail.

THE COST OF CAPITAL

A firm's **cost of capital** is the **average rate it pays its investors** for the use of their money.

Capital budgeting theory is based on the time value of money and the idea of return on investment. A central concept in the theory is the idea of a firm's *cost of capital*. This is the rate of return the firm pays to its long-term investors for the use of their money.

The purpose of the concept is intuitively obvious: An investment makes sense only if it earns more than the cost of funds put into it.

For example, suppose you want to start a business in which you expect to earn a return of 15% on invested money. Further suppose you have no money of your own, but you can borrow from a relative who demands 18% interest. Does it make sense to start the business? Of course not; you'd be losing money from the outset because you'd have to pay more for your funds than you could earn using them. It would make sense to begin the business only if you could borrow the start-up money at *less than* 15%. In this illustration, the cost of capital is the rate at which you can borrow to undertake the venture.

In general, firms have two sources of capital—equity and debt—and pay different rates of return to the investors who supply each. In practice, the cost of capital is a single rate that reflects the average of the rates for those two sources.

Here's a simple example. Suppose the total dollar amount of a firm's capital is 75% equity and 25% debt. Assume the stockholders (equity) are receiving a 10% return, while the creditors (debt) are getting 8% interest. The cost of capital is the *weighted average* of the two returns, where the weights are the proportionate amounts of money invested in each of the two kinds of capital. The calculation follows.

	Portion		Return	
Equity	.75	×	10%	= 7.5%
Debt	.25	×	8	= 2.0
Weighted average cost of capital				= 9.5%

In other words, the cost of capital is a blending of the rates the company provides to its investors. The idea seems simple enough, but can get very complicated in practice.[1] It's an important concept, and we'll devote an entire chapter to it later in the book. For now we'll assume that every firm knows what its cost of capital is, and measures opportunities against it.

CAPITAL BUDGETING TECHNIQUES

In what follows, we'll look at four capital budgeting techniques. Each consists of a series of calculations and a set of decision rules. Using any technique involves calculating a number that the technique associates with a project and then applying the decision rules to that number. Each technique has slightly different decision rules for stand-alone and mutually exclusive situations.

PAYBACK PERIOD

The simplest capital budgeting technique is the payback period. In it we calculate the amount of time it takes for a project's planned cash flows to "pay back" the initial outlay. In other words, we measure the time it takes for the project to break even. This time period is the parameter used for making comparisons. Payback is most meaningful when there's just one cash outflow at the beginning of the project. The technique is most easily understood through a numerical illustration.

In the following table, an initial outlay of $200,000 precedes four inflows of $60,000. The payback period is easily visualized by displaying the cumulative cash flow below the yearly cash flows.

	Year				
	0	*1*	*2*	*3*	*4*
Cash flow (C_i)	$(200,000)	$ 60,000	$ 60,000	$ 60,000	$60,000
Cumulative cash flow	(200,000)	(140,000)	(80,000)	(20,000)	40,000

Payback period = 3.33 years

Notice that the cumulative cash flow is negative by $20,000 after three years and positive by $40,000 after four years. If cash is assumed to flow evenly throughout the year, breakeven occurs after 3.33 years—that is, after three years and four months. This length of time is the **payback period** for the project.

The **payback period** is the time it takes to recover early cash outflows. **Shorter** paybacks are preferred.

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1. We've oversimplified for the sake of this preliminary illustration. As we'll see in Chapter 13, the cost of debt must take into account the tax deductibility of interest.

Payback Decision Rules

Stand-Alone Projects

Decision rules in the payback technique are based on the idea that it's better to recover invested money sooner than later. Companies that use the technique generally have stated policies for the maximum time allowable for capital recovery. The stand-alone decision rule is simply that an acceptable project's payback period must be less than that policy maximum. We can state that rule conveniently as follows.

$$\text{payback period} < \text{policy maximum} \longrightarrow \text{accept}$$
$$\text{payback period} > \text{policy maximum} \longrightarrow \text{reject}$$

Mutually Exclusive Projects

By the same reasoning, we generally prefer a project that pays back sooner to one that pays back later. Therefore, the mutually exclusive decision rule for payback is simply *shorter is better*. If P/B_A and P/B_B represent the payback periods for projects A and B, respectively, we can write the decision rule like this.

$$P/B_A < P/B_B \longrightarrow \text{choose project A over project B}$$

Weaknesses of the Payback Method

Payback is a generally unsophisticated approach to capital budgeting that is criticized for two major shortcomings. First, it ignores the time value of money. Thus, future dollars are weighted equally with current dollars in the calculations. This is clearly a distortion of values. Second, it ignores cash flows after the payback period. This deficiency can lead to the wrong answer even in simple cases.

Example 10.1 Use the payback period technique to choose between mutually exclusive projects A and B.

	Project A	Project B
C_0	$(1,200)	$(1,200)
C_1	400	400
C_2	400	400
C_3	400	350
C_4	200	800
C_5	200	800

SOLUTION: Project A's payback period is clearly three years, because its initial $1,200 investment is entirely recovered in that time. Project B is identical to A for the first two years and only slightly different in the third year, when the cash inflow is just $50 lower. However, that slightly lower payment in year 3 means that B's payback isn't complete until sometime in the fourth year. In other words, the payback period is a little longer for project B than for project A. The payback decision rule therefore chooses A over B. But B is clearly the better project because of what goes on after year 3 when B receives much larger cash inflows. The differences in years 4 and 5 overwhelm the minor difference in year 3 but are ignored by the method.

Why Use the Payback Method?

It's reasonable to ask why anyone uses the payback method given these weaknesses. The answer is that the method is quick and easy to apply, and it serves as a rough screening device.

If a project flunks payback, it's likely to be dismissed without further consideration. If it passes, one of the more sophisticated methods will be applied in further analysis.

The Present Value Payback Method

A variation on the method attempts to correct one of its deficiencies, the fact that it ignores time value. In this approach the payback calculation is made after taking the present value of all the cash flows at an appropriate discount rate.

This approach makes logical sense but still leaves the second deficiency unaddressed while losing the method's "quick and dirty" simplicity. As a result, it's rarely used.

NET PRESENT VALUE (NPV)

A project's **NPV** is the sum of the present values of its cash inflows and outflows at the **cost of capital**.

A fundamental principle of finance and economics is that the *present value* of future cash flows is what counts when making decisions based on value. The **net present value** technique applies this idea to the analysis of projects.

In the net present value technique, we calculate the present value of each of a project's cash flows and add them together. The result is the net present value of the *project*, usually referred to as the *NPV*. The word "net" implies an offsetting of pluses against minuses which reflects the fact that some flows are positive (inflows) and others are negative (outflows). Decisions about which projects to undertake are then based on project NPVs.

The present value calculations are made using the firm's *cost of capital* as an interest rate. This is an important point; the appropriate discount rate for most NPV calculations is the firm's cost of capital as described earlier in this chapter.

We can represent a project's NPV with the following equation.

$$\text{(10.1)} \qquad \text{NPV} = C_0 + \frac{C_1}{(1+k)} + \frac{C_2}{(1+k)^2} + \cdots + \frac{C_n}{(1+k)^n}$$

where C_i represents the project's cash flow in the ith year and C_0 reflects the initial outlay. Notice that we're representing the present value calculation as division by powers of $(1+k)$.[2] Also notice that C_0 isn't divided by anything. That's because the initial cash flow is assumed to occur immediately, in the present.

It's important to think about the signs of the C's in equation 10.1, remembering that negatives are cash outflows and positives are inflows. As we said earlier, the negatives tend to occur first, followed by the positives. Equation 10.1 says that the NPV is the difference between the present values of all the positives and all the negatives. If the present value of the inflows (positives) is greater, NPV is a positive number. NPV is negative if the present value of the outflows is larger.

NPV and Shareholder Wealth

An insightful way to look at capital spending projects involves their impact on shareholder wealth. A project's net present value is the net effect that its undertaking is expected to have on the value of the firm. If a positive-NPV project is taken on and

2. Review equations 6.5 through 6.7 on page 228 if it isn't clear to you that this is equivalent to multiplying by the present value factors $PVF_{k,1}$, $PVF_{k,2}$, and so on.

successfully completed, the economic value of the firm should be increased by exactly the amount of the project's NPV. Conversely, a negative-NPV project will decrease the value of the firm by the amount of the negative NPV.

Therefore, a capital spending program that maximizes the NPV of projects undertaken will contribute to maximizing shareholder wealth, the ideal goal of management. This direct link to shareholder wealth maximization makes NPV the most theoretically correct capital budgeting technique.

So far we've been discussing NPVs for projects to be undertaken in the future. It's important to realize that cash flows may not actually turn out as expected. So a project that has a high *planned* NPV may turn out to have a very different impact on shareholder wealth after it's completed. Nevertheless, before the fact, the planned NPV is our best estimate of the future outcome. We'll talk a lot more about the differences between planned and actual numbers in the next chapter.

Decision Rules

Stand-Alone Projects

A **positive NPV** implies an acceptable project on a **stand-alone** basis.

Clearly, a project in which the present value of planned cash inflows exceeds the present value of outflows is desirable. Conversely, one in which the outflows are larger is undesirable. These situations correspond to projects with positive and negative NPVs, respectively. This logic leads to the *stand-alone decision rule*.

$$NPV > 0 \longrightarrow \text{accept}$$
$$NPV < 0 \longrightarrow \text{reject}$$

Example 10.2 Project Alpha has the following cash flows.

C_0	C_1	C_2	C_3
$(5,000)	$1,000	$2,000	$3,000

If the firm considering Alpha has a cost of capital of 12%, should the project be undertaken?

SOLUTION: Project Alpha's NPV is found by summing the present value of each of the cash flows at the firm's cost of capital. We'll calculate the present values by multiplying each cash flow by the present value factor for 12% and one, two, or three years, respectively. This is equivalent to dividing by powers of $(1 + k)$ as shown in equation 10.1. (See equations 6.5, 6.6, and 6.7 on page 228.) Remember that C_0 isn't factored because it's a present cash flow.

Year	Cash Flow	PV Factor	PV of Cash Flow
0	$(5,000)	1.0000	$(5,000.00)
1	1,000	.8929	892.90
2	2,000	.7972	1,594.40
3	3,000	.7118	2,135.40
		NPV = $	(377.30)

Project Alpha's NPV is negative at the firm's cost of capital, so it should not be undertaken.

It's important to notice that even though a project's total inflows exceed its outflows, it can still have a negative NPV. The reason is that the inflows are generally further in the future, so the present valuing process diminishes their value more than it does that of the outflows. In the example, the undiscounted value of the inflows adds to $6,000, while the single outflow is only $5,000. Nevertheless, on a present value basis, the inflows are less than the outflows.

Mutually Exclusive Projects

In a **mutually exclusive** context, projects with **larger NPVs** are **preferred**.

The more the present value of cash inflows exceeds outflows, the larger is a project's NPV and the more it can be expected to contribute to shareholder wealth. In other words, *a bigger NPV is better than a smaller NPV.* This leads to the *mutually exclusive decision rule.*

$$\text{NPV}_A > \text{NPV}_B \longrightarrow \text{ choose project A over B}$$

where NPV_A and NPV_B are the net present values of projects A and B, respectively.

The idea is straightforward on its face: Choose the project with the largest NPV. However, several questions come up in actual practice. The following example provides another drill on calculating NPVs and applying the decision rules; at the same time it raises some issues related to the practical application of the method.

Example 10.3

The Xavier Motor Company makes outdoor power equipment including lawn mowers and garden tractors and is considering two diversification ventures. The first involves manufacturing a larger, more powerful tractor than the firm has made up until now. Market research indicates a substantial demand for more powerful equipment, and some competitors are already moving in that direction. The second opportunity involves building snowblowers. The manufacturing and engineering technology required for making snowblowers is essentially the same as that for building garden equipment, but Xavier has never made snowblowers before.

Management wants to make a decision based on only five years of projected cash flows, because it feels the future beyond that time is too vague to form a basis for current decisions. In other words, if a project isn't expected to earn enough to justify itself in five years, management considers it too risky.

Working with representatives from the marketing, engineering, and manufacturing departments, a financial analyst has put together a set of projected incremental cash flows for each project. Xavier's cost of capital is 9%.

Xavier Motor Company Project Estimates ($000)

Year	Tractor	Snowblower
0	$(3,000)	$(3,500)
1	(250)	(700)
2	500	800
3	1,000	1,200
4	1,500	2,000
5	1,500	2,000

A financial analysis of the project situation should provide answers to the following questions.

a. If these projects were being considered on a stand-alone basis, would either or both be acceptable?

b. If Xavier can raise no more than $5 million for new projects, which of these projects should be chosen?
c. If Xavier's management is willing to consider two more years of projected cash flow, and the contributions continued at the level of the last two years, which project would be chosen?
d. Are any risk considerations relevant beyond the numbers in this situation?

SOLUTION: We begin by calculating the NPV for each project. We'll show these calculations in tabular form, multiplying the yearly cash flows by the present value factor for 9% and the appropriate year ($PVF_{9,i}$), rather than dividing by powers of $(1 + k)$.

Xavier Motor Company NPV Project Analysis ($000)

Year	Factor	Cash Flows		PV of Cash Flows	
		Tractor	*Snowblower*	*Tractor*	*Snowblower*
0	1.0000	$(3,000)	$(3,500)	$(3,000)	$(3,500)
1	.9174	(250)	(700)	(229)	(642)
2	.8417	500	800	421	673
3	.7722	1,000	1,200	772	927
4	.7084	1,500	2,000	1,063	1,417
5	.6499	1,500	2,000	975	1,300
				NPV = $ 2	$ 175

We can answer the first two questions immediately from the foregoing calculations.

a. Both projects have a positive NPV and are therefore acceptable on a stand-alone basis. However, neither is positive by very much in relation to the size of the investments involved. The tractor project is especially marginal. This result is bound to raise some questions about the advisability of the projects. For now we'll just note that the projects are acceptable in accordance with the NPV method and return to the issue of accuracy later.
b. The projects are mutually exclusive, because their initial outlays total $6.5 million and the company only has $5.0 million available for capital projects. In the mutually exclusive situation, the snowblower appears to have the edge, but not by much.

The next two questions require a few more calculations and a lot more judgment.

c. The distant future is always hard to pin down. It's easy to forecast great sales and profitability six or more years in the future. Exuberant sales and marketing people do it all the time. The question is, how much of those forecasts should a reasonably prudent financial manager accept when making decisions about the commitment of substantial amounts of money? Let's calculate the impact of another two years on the NPV analysis.

Xavier Motor Company NPV Project Analysis ($000)

Year	Factor	Cash Flows		PV of Cash Flows	
		Tractor	*Snowblower*	*Tractor*	*Snowblower*
6	.5963	$1,500	$2,000	$ 894	$1,193
7	.5470	1,500	2,000	821	1,094
			Addition to NPVs	1,715	2,287
			Previous NPVs	2	175
			New NPVs	$1,717	$2,462

Notice how the entire complexion of the problem has changed. Both projects now clearly appear favorable on a stand-alone basis. That is, the NPVs are substantially positive relative to the size of the early investments. Further, the snowblower now seems to be an obvious choice with a substantial NPV advantage. It's possible to make virtually any project look good by forecasting positive cash flows in the distant future.

d. *This question raises another big issue: Are the forecasts for the two projects equally reliable? There's a strong argument that they are not. Snowblowers are a new business for Xavier, while the tractor is an extension of something it's already doing. The implication is that the snowblower project may be much riskier than the tractor project. If that's the case, is a simple comparison of the NPVs valid? Probably not.*

> Capital budgeting techniques are easy to apply, but **interpreting** the results requires considerable judgment.

For now you should concentrate on being sure you understand the mechanics of the NPV method—that is, how to calculate NPVs and how to apply the decision rules. However, keep the concerns brought up by this problem in mind. We'll return to them in the next two chapters.

INTERNAL RATE OF RETURN (IRR)

Instead of comparing present value dollar amounts, the **internal rate of return** (IRR) technique focuses on rates of return. The IRR concept can be defined in two ways. In fundamental terms a project's IRR is just the return it earns on invested funds. However, the concept can also be defined in terms of the NPV equation, 10.1. We'll have a close look at both approaches.

The Project as an Investment

In the IRR method we view a project as an investment of the company's money, which in principle is similar to the purchase of a financial asset. In this view, the cash outlay at the beginning of a project is like an investor paying cash to purchase a stock or a bond. Subsequent cash inflows from the project are similar to interest or dividend payments received by the investor.

The analogy is easy to visualize when a project has only one cash outflow occurring at its beginning, time zero. Consider the project in Example 10.2; we'll repeat those cash flows here for convenience.

C_0	C_1	C_2	C_3
$(5,000)	$1,000	$2,000	$3,000

> A project's **IRR** is the return it generates on the **investment** of its cash outflows.

Notice that the project calls for one cash outflow, or payment, followed by three inflows. The initial $5,000 outlay can be thought of as the "price" of receiving the subsequent inflows. In other words, accepting the project financially amounts to putting up the initial $5,000 in return for which the investing company receives the later inflows.

Recall that in Chapters 7 and 8 we defined the return on an investment as the interest rate at which the discounted value of the future cash flows just equals the price of the investment (see pages 271 and 329). The same idea applies here. The IRR is the interest rate at which the present value of the three inflows just equals the $5,000 outflow (price).

In this view, the IRR is analogous to the yield on a bond. Recall that purchasing a bond entitles the owner to a series of interest payments and a repayment of principal. These are cash inflows for which the investor pays the bond's price. The interest rate that equates the present value of the investor's inflows to the price of the bond (outflow) is the bond's yield. Similarly, the rate that equates the present value of a project's inflows to the initial outlay is the IRR.

The idea is a little harder to see when there's more than one outflow—for example, if there are two negative cash flows before the inflows start. In such a case, the IRR is the interest rate that equates the present value of all inflows with the present value of all outflows. That is, by definition, the IRR is just the return on the investment of the outflows.

Defining IRR through the NPV Equation

We can gain additional insight by relating the IRR concept to the NPV method. The two approaches are closely related in that both NPV and IRR can be defined by essentially the same equation.

In the last section we defined a project's NPV with equation 10.1. Referring to that expression, IRR is simply the value of the interest rate, k, at which NPV equals zero. This occurs when the present value of all the inflows is just equal to the present value of all the outflows, and they offset one another. This means IRR is the solution to the NPV equation with the interest rate treated as the unknown and NPV set to zero. This is the same as saying that *the IRR is the interest rate at which a project's NPV equals zero*.

Rewriting 10.1 with NPV = 0 and using IRR in place of k yields the expression that defines IRR.

> The **IRR** is the interest rate that **makes a project's NPV zero**.

$$\textbf{(10.2)} \qquad 0 = C_0 + \frac{C_1}{(1 + IRR)} + \frac{C_2}{(1 + IRR)^2} + \cdots + \frac{C_n}{(1 + IRR)^n}$$

When the C_i are given for a particular project, equation 10.2 is one equation in one unknown, IRR. The solution is *the* IRR for the project. Every project of practical interest has an IRR just as every project has an NPV.

Decision Rules

IRR decision rules follow directly from thinking in terms of a return on an investment.

Stand-Alone Projects

In the stand-alone case, we're asking whether investment in a project is a good use of the company's money. The answer depends on the rate the firm pays to use that money. We described that rate as the company's *cost of capital* earlier in this chapter.

Recalling the illustration given there (starting a business with borrowed money, page 423), we can generalize by saying that no one should invest in anything unless the return on the investment is expected to exceed the rate paid for the use of the money invested.

> A project is acceptable on a stand-alone basis if its **IRR exceeds the cost of capital**.

Because a project's IRR is the return on funds invested in the project, and the cost of capital reflects the average rate the company pays for the use of long-term money, the stand-alone decision rule follows from this generalization: *Invest in a project only if its IRR exceeds the firm's cost of capital*. Or,

$$IRR > k \longrightarrow \textbf{accept}$$
$$IRR < k \longrightarrow \textbf{reject}$$

where k is the firm's cost of capital.

Mutually Exclusive Projects

In a **mutually exclusive** context, projects with **larger IRRs** are **preferred**.

The decision rule for mutually exclusive projects also follows from the definition of IRR as a return on an investment. We prefer investments with higher rates of return to those with lower rates. Hence, *a bigger IRR is better*. Or, if IRR_A and IRR_B relate to projects A and B, respectively,

$$IRR_A > IRR_B \longrightarrow \text{ choose project A over project B}[3]$$

Calculating IRRs

Examination of equation 10.2 shows that calculating internal rates of return for a general series of cash flows isn't easy. A project's IRR is the solution to that equation when a fixed set of numbers has been substituted for the C_i. Notice that the equation is a polynomial of order n in the variable IRR. Further, it's a very messy polynomial, because the powers of the unknown appear in the denominators of the fractions on the right. In general, such an equation can't be solved algebraically for values of n greater than 2.

We get around this difficulty by using an iterative, numerical approach to solving the equation. In fact, we actually use equation 10.1 from which 10.2 is derived to find a solution to the latter.

Equation 10.1 defines a project's NPV and enables us to calculate that figure for any interest rate, k, given a set of cash flows (the C_i). To find a project's IRR, we simply try different values for k in equation 10.1 until we find one at which the NPV is zero. That value of k is the IRR, and at that value equation 10.1 *becomes identical* to 10.2.

Finding IRRs usually requires an **iterative, trial-and-error** technique.

To solve a problem, we guess at the project's IRR and calculate an NPV using the guess as the interest rate in equation 10.1. If the NPV doesn't come out to be zero, the first guess was incorrect, and we guess again. However, the result of the first calculation contains information that indicates the direction in which the second guess should be made. An illustrative example will make the procedure clear.

Example 10.4 Find the IRR for the series of cash flows in Example 10.2.

C_0	C_1	C_2	C_3
$(5,000)	$1,000	$2,000	$3,000

If the firm's cost of capital is 8%, is the project a good idea? What if the cost of capital is 10%?

SOLUTION: We'll start by guessing that the IRR is 12% and calculating the project's NPV at that rate. As it happens, we've already done that calculation in Example 10.2. Review that calculation on page 427 now and see that the resulting NPV is ($377.30).

Clearly, the project's NPV at 12% is not zero, so we have to make another guess. To focus that guess, look at the problem's pattern of cash flows. The positive numbers are in the future, displayed on the right. These positives are affected by the discounting process when we take present values. In effect, they're shrunk by their respective present value factors before being combined with the negative $5,000 to form the NPV. *Notice that a larger interest rate shrinks the positive numbers more than a smaller rate does, but doesn't affect the initial outlay, because it isn't discounted.*
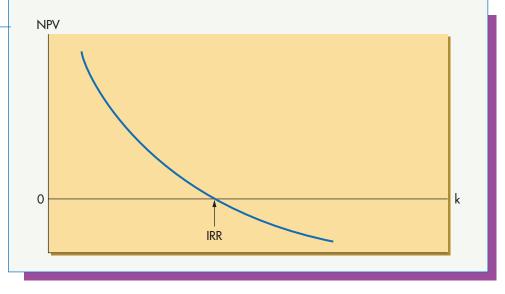
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3. We'll see shortly that the IRR decision rule can occasionally lead to the wrong choice in mutually exclusive situations.

Our first guess of 12% shrank the positive numbers too much, so that they became less than the negative outlay of $5,000 by $377.30. We'd like our next guess to shrink the positive cash flows less, so we'll choose a *smaller* interest rate.

We can summarize this thinking by saying that the magnitude of a project's NPV moves inversely with the interest rate used in its calculation. This relationship is portrayed graphically in Figure 10.1.

Figure 10.1

NPV Profile

The graph depicted is known as an *NPV profile*. Every project with a defined set of cash flows (the C_i) has an NPV profile that appears as a line on the graph. The lines will generally be downsloping to the right and cross the horizontal axis at some point. The IRR is the point at which the project's NPV is zero. That occurs where the NPV profile crosses the k axis as shown in the graph.

A project's **NPV profile** is a graph of its NPV versus the cost of capital. It **crosses** the horizontal **axis** at the **IRR**.

It's important to realize that the NPV profiles of capital budgeting projects slope downward to the right, because the pattern of cash flows generally involves outflows first (negative numbers) and inflows later (positive numbers). A higher discount rate therefore affects the positives more than the negatives, and has the net effect of shrinking the NPV. The analysis of unusual patterns is left as an exercise.

Finding a project's IRR is equivalent to locating the crossover point of the NPV profile and the horizontal axis by testing points on either side. In the current problem, our first guess has taken us to the right of the crossover. Our next guess must be a lower interest rate to move the NPV up and get closer to the IRR.

We'll keep track of our calculations by setting up a two-column table to portray each interest rate choice and the NPV calculated to go along with it. Use the calculation method shown in Example 10.2 to verify that the entries shown below are correct. NPVs are shown rounded to the nearest whole dollar.

Interest Rate Guess	Calculated NPV
12%	$(377)
10	(184)
9	(83)
8	22
7	130

The calculated NPV changes sign between 9% and 8%, which means the IRR is between those interest rates.

If the firm's cost of capital is 8%, the project is marginally favorable. If the cost is 9%, it's clearly unfavorable.

Notice that this technique is similar to the iterative approach we used to find the yield on a bond given its price in Chapter 7 (pages 281–283).

Financial calculators are programmed to find the IRR for a general set of cash flows. Internally, such machines are going through the iterative process we've just illustrated.

Technical Problems with IRR

Two technical problems are associated with the IRR method. They *rarely* present practical difficulties, but anyone using the technique should be aware that they exist.

Multiple Solutions

The IRR for a project is defined as the solution to equation 10.2 with the project's cash flows substituted for the C_i and IRR treated as the unknown. The expression comes from the NPV equation operating at the point where NPV equals zero.

A problem arises because equation 10.2 is an nth-order equation in the variable IRR, where n is the number of years the project lasts. That means it can have as many as n solutions. How do we know which one is correct? At first glance, this seems like a fatal problem for the IRR method, but in reality it isn't.

Solutions to the equation can be either positive, negative, or what mathematicians call imaginary.[4] Negative and imaginary solutions aren't practically meaningful, so we needn't be concerned about them. We worry only when there's more than one positive solution.

It turns out that the number of positive solutions to equation 10.2 depends on the pattern of the project's cash flows. There can be no more positive solutions than there are *sign reversals* in the cash flow stream. A sign reversal occurs when the C_i change from negative to positive or from positive to negative.

The normal pattern of project cash flows involves only one sign change. There's almost always a negative initial outlay, C_0, followed by a series of periods with positive cash flows. C_0 is sometimes followed immediately by a few negative flows before the inflows start, but even then there's only one sign change from negative to positive. That means there's only one positive solution for IRR, which is the correct one.

In the occasional project with one or two negative cash flow years interspersed among the positives, there can be more than one IRR solution. However, in practical problems there's generally only one solution within a reasonable range of values for an interest rate, say, between 0% and 50%. That's the one we're looking for. When other positive solutions exist, they tend to be figures like 300% or 400%. Because we solve for the IRR by using an iterative approach starting with a reasonable guess, we rarely find such answers.

As a practical matter, the multiple-solution issue can be all but ignored.

The Reinvestment Assumption

Examine equation 10.2 once again. Suppose we have a typical case with a negative C_0 followed by a long series of cash inflows. The IRR method makes an implicit assumption about what happens to those cash inflows after they're received. It assumes that inflows are *reinvested* at the IRR until the end of the project's life.

Unusual projects can have **more than one IRR**, but they rarely present practical difficulties.

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4. Imaginary numbers are functions of the square root of minus one (-1). They are the subject of an entire branch of advanced mathematics.

The reinvestment assumption presents a problem in the case of especially profitable ventures. Suppose a project has an IRR of 50%. The company is unlikely to find other opportunities with returns that high in which to reinvest the funds thrown off by the project. Therefore, the reinvestment assumption is unlikely to be satisfied. But that casts a doubt on the reality of the 50% solution. In other words, the return rate is very high, but it may not truly be 50%.

Contrast IRR with the NPV method in such a situation. A project with a very high IRR would also have a high NPV, but the reinvestment assumption in that method requires only that cash flows be reinvested at the cost of capital. That's because the discount rate in the NPV technique is just k, the cost of capital, as shown in equation 10.1. Such investments are virtually always available.

The reinvestment problem is also somewhat academic—that is, not a practical concern. When returns on projects are in the 50% neighborhood, people don't worry about *exactly* how high they are. In other words, if a project computes to a 50% IRR, people don't argue about whether it's 50% or only 40%. In either case, if the projected cash flows are correct, they indicate a very good opportunity.
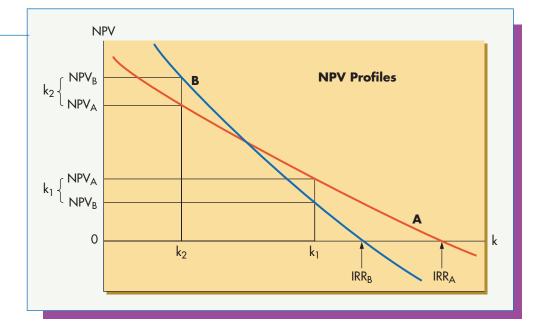
COMPARING IRR AND NPV

The internal rate of return and the net present value methods are the two major approaches to evaluating capital budgeting projects. It's logical to ask whether they always give the same solutions to problems. Surprisingly, the answer to that question is no. Let's explore why with the aid of the NPV profile that we introduced earlier.

The NPV profile *for a project* is a graphic representation of the relationship between a project's NPV and the interest rate at which it's calculated. It is simply the graph of equation 10.1 for a particular set of cash flows (the C_i).

Look back at the NPV profile in Figure 10.1. The related discussion demonstrated that the line of the profile slopes downward to the right for projects that involve early cash outflows and later cash inflows. Because that is invariably the case for business projects, all profiles of interest to us slope downward to the right. However, their shapes aren't identical, and they cross the horizontal axis at different points. In addition, the profiles of different projects *may* cross one another, as illustrated in Figure 10.2.

Figure 10.2

Projects for Which IRR and NPV Can Give Different Solutions

We can use the NPV profiles depicted there to show how the NPV and IRR methods can give conflicting directions when we're choosing among mutually exclusive projects.

Notice in Figure 10.2 that project A's profile crosses the k axis to the right of project B's. That means $IRR_A > IRR_B$. Therefore, the IRR method chooses project A over project B. Does the NPV method also choose project A? It depends on the cost of capital. We'll demonstrate by graphically evaluating the two projects against each other using the NPV method at different values of k.

To determine graphically any project's NPV at a particular cost of capital, we find k on the horizontal axis and move vertically to the project's NPV profile. From there we move left to the vertical axis and read the value of NPV.

First choose cost of capital k_1 and locate NPV_A and NPV_B, the NPVs for projects A and B, respectively. These are shown toward the bottom of the vertical axis in the diagram. Notice that NPV_A is above (is larger than) NPV_B, indicating that the NPV method chooses project A just as the IRR method does.

Now do the same thing for cost of capital k_2, which is to the left of the point at which the two profiles cross. The resulting NPVs are shown toward the top of the vertical axis. Notice that the result is reversed. This time the NPV method chooses project B over A, a result opposite that of the IRR method.

Examination of the diagram provides some insight into when this phenomenon may occur. To give conflicting results, the NPV profiles have to cross in the first quadrant of the graph at interest rates that are of practical interest. That doesn't happen often. Even when it does, we can generally expect the two methods to agree when the cost of capital isn't too far below the project IRRs.

The NPV and IRR methods can occasionally give **conflicting results** in mutually exclusive decisions.

The Preferred Method in Case of Conflict

As a practical matter, conflicts between the IRR and NPV methods are rare. When they do occur, *the NPV method is preferred*, because its *reinvestment assumption* is more easily satisfied.

NPV AND IRR SOLUTIONS USING FINANCIAL CALCULATORS AND SPREADSHEETS

Modern financial calculators and spreadsheet software take most of the drudgery out of calculating NPVs and IRRs. The machines are especially convenient when we need to find an IRR because they allow us to avoid the tedious iterative process described earlier.[5] We'll outline how to solve problems using a Texas Instruments BAII PLUS™ calculator and Microsoft's Excel™ spreadsheet. Inputting to the calculator can be a little tricky, so you'll probably have to refer to its user's manual as well.

Calculators

A calculator will find a project's NPV or IRR quickly once the associated cash flows are entered. To enter cash flows on the BAII PLUS, begin by pressing the CF button to enter the cash flow mode, and then clear the working memory by pressing 2nd and then CLR Work.

The calculator is programmed to prompt the user for the cash flows one at a time, starting with the initial outlay that it calls CFo (we've called this figure C_0). Type in the value, make it negative (an outflow) by hitting the +/− key, and then press ENTER. (After entering a number, press the ↓ key to move to the prompt for the next input item.)

---

5. It's worth noting that the machines are programmed to find IRRs using a similar iterative process.

The machine then prompts for up to 24 *different* cash flows, referred to as C01, C02, . . . , C24. However, after each of these cash flows is entered, the machine prompts for a "frequency" displayed as F01, F02, . . . , F24. The frequency allows you to repeat the last cash flow entered up to 9,999 times before moving on to the next *different* cash flow. If you don't enter a frequency, the machine assumes 1. For example, the series

$$-500 \;\; 100 \;\; 200 \;\; 200 \;\; 200 \;\; 800$$

can be entered with the following inputs.

Prompt	Entry
CFo	−500
C01	100
F01	
C02	200
F02	3
C03	800

You can move back and forth through the cash flow and frequency figures you've entered using the ↑↓ keys. Cash flows can also be inserted, deleted, or changed using procedures outlined in the user's manual.

NPV and IRR

Once a project's cash flows are in the machine, calculating NPV and IRR is easy. To find the NPV, begin by pressing the NPV button. The machine will prompt for I, the interest rate at which you want to do the present value calculation (this is generally the cost of capital we've called k). Type the interest rate as a whole number (e.g., 12 for 12%) and press ENTER. Now press ↓ and then CPT (for compute). The project's NPV will appear on the screen. To get IRR, just press IRR and then CPT.

As an exercise, use your calculator to compute NPVs and IRRs for the project cash flows in Examples 10.2 and 10.3.

Spreadsheets

Spreadsheet solutions for NPV and IRR are very easy to do. We simply arrange the project's cash flows in a series of consecutive cells along a row or column and use the spreadsheet software's NPV and IRR functions. Here's an example using Microsoft Excel^{TM} and the cash flows from Example 10.2.

	A	B	C	D	E
1	**Project Cash Flows:**				
2		−$5,000	$1,000	$2,000	$3,000
3					
4	**Cost of Capital: k =**		0.12		
5					
6	**NPV =**	($377.41)			
7					
8	**IRR =**	8.2%			

The formula in cell B6 is

$$= B2 + NPV(C4, C2{:}E2)$$

Let's first focus on the NPV function. Its first argument is an interest rate—the cost of capital in this case—which we've put in cell C4. Notice that we input interest rates (cell C4) in decimal form in spreadsheets rather than as whole numbers as we do when using calculators. The other argument is the range of cells containing the project's *future* cash flows, C2 to E2. Notice that the NPV function calculates the present value of the *future* cash flows only. That means we have to add the initial outlay separately, which we do by including B2 in the formula.

The IRR function, on the other hand, takes the whole series of cash flows including the initial outlay. The formula in cell B8 is simply

$$= \text{IRR(B2:E2)}$$

PROJECTS WITH A SINGLE OUTFLOW AND REGULAR INFLOWS

Many projects are characterized by a single initial cash outflow followed by a finite number of equal inflows coming at regular time intervals. As an illustration, take the project in our last example and shift $1,000 from the third year to the first.

C_0	C_1	C_2	C_3
$(5,000)	$2,000	$2,000	$2,000

Annuity formulas can be used to calculate NPV and IRR when projects have **single outlays and regular inflows**.

This pattern is much easier to work with because the inflows can be treated as an annuity. In such cases, we can rewrite the equations, defining NPV and IRR using the present value of an annuity formula.[6]

Equation 10.1 defining NPV becomes

(10.3) $$\text{NPV} = C_0 + C[\text{PVFA}_{k,n}]$$

where C is the constant annual inflow, k is the cost of capital, n is the project's life in years, and C_0 is the initial outlay. Remember that C_0 is a negative number. The second term on the right is the present value of the annuity formed by the project's positive cash flows over a period of n years at interest rate k.

Similarly, equation 10.2 defining IRR becomes

(10.4) $$0 = C_0 + C[\text{PVFA}_{\text{IRR},n}]$$

Equation 10.4 is especially convenient, because it lets us avoid the iterative procedure otherwise necessary to find IRRs.

Example 10.5 Find the NPV and IRR for the project used as an illustration immediately above. Assume the cost of capital is 12%.

SOLUTION: To calculate NPV, write equation 10.3 and substitute from the cash flow pattern.

$$\text{NPV} = C_0 + C[\text{PVFA}_{k,n}]$$

$$\text{NPV} = -\$5,000 + \$2,000[\text{PVFA}_{12,3}]$$

---

6. We're using the present value of an annuity from Chapter 6, page 243; but replacing PMT with C just to be more consistent with our present notation, which represents cash flows as C's.

Find the present value factor for an annuity in Appendix A-4 and substitute

$$NPV = -\$5,000 + \$2,000(2.4018)$$
$$NPV = -\$196.40$$

To calculate the IRR, write equation 10.4 and substitute

$$0 = C_0 + C[PVFA_{IRR,n}]$$
$$0 = -\$5,000 + \$2,000[PVFA_{IRR,3}]$$

Solve for the factor.

$$PVFA_{IRR,3} = \$5,000/\$2,000$$
$$= 2.5000$$

Now search for 2.5000 in Appendix A-4 along the row for three periods. The solution is between 9% and 10%.

Compare these results for NPV and IRR with those we calculated in Examples 10.2 and 10.4. Did shifting the $1,000 forward by two years make a big difference?

PROFITABILITY INDEX (PI)

The **profitability index (PI)** is the ratio of the present value of inflows to the initial outlay. Projects are **acceptable if PI > 1. Larger PIs are preferred**.

IRR and NPV are the most widely used capital budgeting techniques. Payback is used frequently but generally as a preliminary screening device before one or both of the other methods. Although new approaches are proposed from time to time, none has caught on in a big way. One approach, however, is used often enough to make it worth mentioning briefly. The **profitability index (PI)** is essentially a variation on the NPV method. We'll define it by referring to equation 10.1.

The PI compares the present value of a project's *future* cash flows with the initial outlay required to get the project started, making the comparison in the form of a ratio.

Recall that the initial outlay is C_0. Hence, PI is defined as the sum of all the terms to the right of C_0 in equation 10.1 divided by C_0.

$$\textbf{(10.5)} \qquad PI = \frac{\dfrac{C_1}{(1+k)} + \dfrac{C_2}{(1+k)^2} + \cdots + \dfrac{C_n}{(1+k)^n}}{C_0}$$

The PI is also known as the benefit/cost ratio, reflecting the idea that the positive cash flows expected in the future are benefits, while the initial outlay is a cost.

The concept is poorly defined if some of the early C's after C_0 are negative. In such a case, it isn't clear whether those should be considered costs and added to the denominator or negative benefits and subtracted from the numerator. The idea works best when the initial outlay, C_0, is the only negative cash flow, which is a fairly common situation. (We should really write $-C_0$ in the formula, but generally don't.)

Essentially, the PI is the ratio

$$\frac{\textbf{present value of inflows}}{\textbf{present value of outflows}}$$

NPV, by way of contrast, is the *difference* between the present value of inflows and the present value of outflows.

When the present value of inflows exceeds the present value of outflows, the PI will be greater than 1.0. This condition is equivalent to a positive NPV. Further, a larger PI is preferable to a smaller PI, because it implies more inflows relative to outflows in a present value sense. This is equivalent to preferring a larger to a smaller NPV.

Decision Rules

All this leads to the decision rules for the profitability index.

Stand-Alone Projects

$$\text{PI} > 1.0 \longrightarrow \text{accept}$$
$$\text{PI} < 1.0 \longrightarrow \text{reject}$$

Mutually Exclusive Projects

$$\text{PI}_A > \text{PI}_B \longrightarrow \text{choose project A over project B}$$

where PI_A and PI_B are the profitability indices for projects A and B, respectively.

Comparison with NPV

The comparison with NPV decision rules isn't exact. In the stand-alone case, a $\text{PI} > 1.0$ always coincides with $\text{NPV} > 0$. However, the two methods may compute the relative desirability of projects differently and may not make the same choices among competing options.

Example 10.6 Compute the profitability index for a project with the following cash flows if the cost of capital is 9%. Is the project acceptable on a stand-alone basis?

C_0	C_1	C_2	C_3
$(4,500)	$1,500	$2,000	$1,600

SOLUTION: The present value of future cash flows is computed as follows:

Year (i)	C_i	$PVF_{9,i}$	PV
1	$1,500	.9174	$1,376
2	2,000	.8417	1,683
3	1,600	.7722	1,236
			$4,295

Then, from equation 10.5, the PI is

$$\text{PI} = \frac{\$4{,}295}{\$4{,}500} = .95$$

Because the profitability index is less than 1.0, the project is not acceptable.

COMPARING PROJECTS WITH UNEQUAL LIVES

Projects with substantially **unequal lives** aren't directly comparable.

Mutually exclusive decisions are sometimes complicated by the fact that the competing projects don't extend over the same period of time. When the disparity is significant, it can make a direct comparison of the projects meaningless.

For example, suppose a manufacturing company is replacing a production machine and must choose between two new models that have different lives. Assume both new

PRACTICAL **FINANCE**

Which Methods Do Businesses Prefer?

Big Companies

Virtually all large companies do capital budgeting using sophisticated, time value-based methods. However, although NPV is theoretically the best technique, companies use IRR more frequently. That's probably because people are more comfortable with rates of return than with sums of present valued dollars. We're all familiar with interest rates on savings accounts from childhood. We learn about returns on stock investments early in our adult lives, and businesspeople talk about returns on sales, assets, and equities all the time.

NPV, on the other hand, is a little abstract. Most people don't learn about it until college, and you can't hold the present value of a dollar in your hand. Executives understand NPV, but the concept is a little alien. As a result most gravitate toward IRR. This preference isn't likely to change much in the foreseeable future.

Small Companies

Things are very different in small businesses. A study done some years ago indicated that more than 80% of small businesses commit capital on the basis of the payback method or don't do any formal analysis at all.
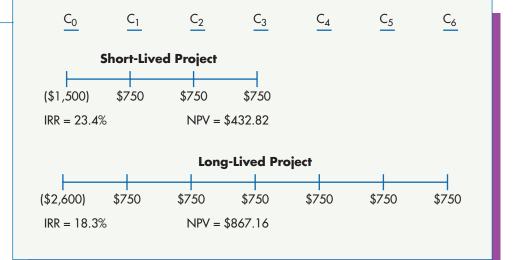
There are probably two reasons for that finding. First, it's likely that most small firm managers simply haven't been educated in finance and don't know how to apply the more sophisticated techniques. Second, the small firm focus is just about always on very short-term cash flow. In the very short run, payback does a creditable job, so entrepreneurs may feel they don't need anything else.

Source: L. R. Runyon, "Capital Budgeting Decision Making in Small Firms," *Journal of Business Research* 11 (September 1983): 389–97

machines save $750 per year in cost, but the longer-lived model is expected to last six years, while the other will be good for only three years. Of course, the more durable replacement will be more expensive. Assume the costs of the machines are $2,600 and $1,500, respectively. A comparison of IRRs and NPVs is shown in Figure 10.3 for an 8% cost of capital.

Figure 10.3

Comparing Projects with Different Lives

Notice that the shorter project has a better IRR, but the longer has a superior NPV. This conflict is due to the disparity in the project's lives.

The problem arises mainly with the NPV method. To visualize the difficulty, think of a replacement machine as having an annual benefit to the firm during its entire life. In broad terms, that just means it will produce income every year in excess of its cost on a present value basis.

The problem here is that the NPV method adds up six years of benefits for one project and only three years for the other. Therefore, the longer-lived machine just about always winds up with a higher NPV.

Putting it another way, the fact that one machine has a six-year life forces us to look at a six-year time horizon for analysis, and the shorter-lived project is implicitly assumed to have nothing going on in the second three years.

The Replacement Chain Method

The **replacement chain method** extends projects until a common time horizon is reached.

To solve the dilemma we have to realize that if the company buys the cheaper machine, it will have to buy another replacement at the end of the first three years.

The correct way to compare the two projects in Figure 10.3 is to explicitly include a replacement for the short-lived machine at the end of its life. In essence, we *chain* two of the short projects together to cover the same time span as the longer project. The idea is portrayed graphically in Figure 10.4. The time line pictured there should replace the short line in Figure 10.3.

Figure 10.4

A Three-Year Project Chained into Six Years

In this example, the long-lived project still has a higher NPV, so it would be chosen. However, the NPV comparison is more reasonable now that the second three-year period is considered in the short-lived case. Notice that the IRR of the short-lived project is not affected by chaining.

In theory, any pair of projects with different lives can be compared using the *replacement chain method* by chaining both until a common time period is reached. For example, a 3- and a 4-year project could be compared by chaining both to 12 years.

The approach has a significant drawback in that a large number of replacements may be necessary to get equal time horizons for the competing projects. For example, if one option lasts 5 years and the other 8, we'd have to look at 40 years of replacements to make the comparison, and that isn't realistic.

The Equivalent Annual Annuity (EAA) Method

Turning each project into an *equivalent annual annuity* (EAA) is the easiest way to solve the time disparity problem. To understand the method, think in terms of chaining

The **equivalent an-nual annuity** method replaces each proj-ect with an equiva-lent perpetuity.

projects the way we did in the last section. However, replace each project link in the chain by its NPV. Then replace that with an annuity of the same length whose NPV is equal to the project's NPV. A numerical example will make the idea clear.

The shorter-lived project we've been working with has a life of three years and an NPV of $432.82 (Figure 10.3). The equivalent three-year annuity is found by substi-tuting that amount into the present value of an annuity formula along with three years and the appropriate interest rate as follows.
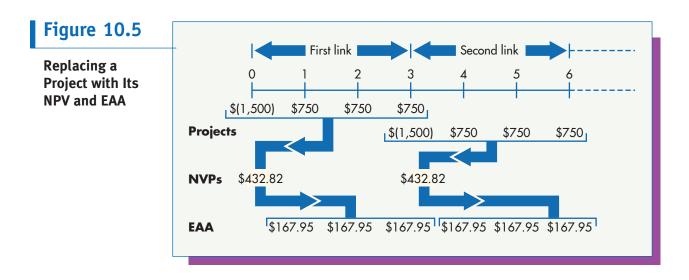
$$PVA = PMT[PVFA_{k,n}]$$
$$\$432.82 = PMT[PVFA_{8,3}]$$

Now find the factor in Appendix A-4, substitute, and solve for the annuity payment, which is the EAA.

$$\$432.82 = PMT(2.5771)$$
$$PMT = \$167.95$$
$$= EAA$$

The idea is illustrated in Figure 10.5 for the three-year project shown chained in Figure 10.4. As the diagram indicates, we make two conceptual steps. First we replace the project with its NPV. Then we replace that with an annuity of the same length which has the same NPV. In this way we replace the project with an equivalent stream of equal benefits.

Figure 10.5

Replacing a Project with Its NPV and EAA

Because we can chain the project forward through time as long as we like, we can represent it by an indefinitely long stream of payments equal to the EAA. It's impor-tant to realize that this is true even though the EAA calculation is based on the number of years in the life of the original project.

Further, we can calculate an EAA for *any* project regardless of its life. And because all EAAs are infinite annuities, we can choose among projects by comparing their annuity payments.

The longer-lived project in our illustration has an NPV of $867.16 and a life of six years. Its EAA is calculated as follows.

$$PVA = PMT[PVFA_{k,n}]$$
$$\$867.16 = PMT[PVFA_{8,6}]$$
$$\$867.16 = PMT(4.6229)$$
$$PMT = \$187.58$$
$$= EAA$$

This is larger than the shorter project's EAA of $167.95, so we again come to the conclusion that the longer project is better.

Notice that we can also compare EAAs on the basis of their present values by using the present value of a perpetuity formula. This simply amounts to dividing the EAA by the decimal value of the relevant interest rate. Because the cost of capital is the same for both projects, this will result in the same choice.

CAPITAL RATIONING

A firm's *capital budget* is the total amount of money to be spent on capital projects in a period of time, usually a year. How large should that amount of money be?

In theory, the answer to that question is easy. Every project with a positive NPV is expected to increase shareholder wealth and should be undertaken. Therefore, the optimal capital budget would be large enough to undertake all available projects with positive NPVs or equivalently with IRRs that exceed the cost of capital.

We'll illustrate the idea by considering a company with the following projects available, sorted in decreasing order of IRR.

Project	IRR	C₀
A	16%	$8M
B	14	5M
C	12	6M
D	11	3M
E	8	6M
F	6	7M

Figure 10.6 plots the projects on a graph that displays interest rates against cumulative capital spending. Each lettered block represents a proposed project. The blocks' heights are the project IRRs while their widths are the amounts of capital each requires to get started. Usually that's the project's initial outlay, C_0.

Notice again that the projects are arranged in decreasing order of IRR, and that the firm's cost of capital is shown as a horizontal line. The projects are either stand-alone opportunities or the best choice among mutually exclusive options.
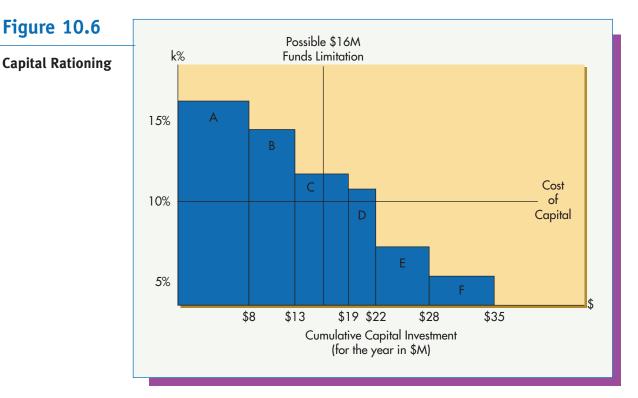
This portrayal makes it easy to see which projects have IRRs that exceed the cost of capital and therefore should be done. In this case, if there is no funds limitation, the firm will maximize shareholder wealth by undertaking projects A, B, C, and D while forgoing projects E and F.

In practice, however, there is rarely enough money available to do all proposed projects that appear to have positive NPVs. Some capital constraint is likely to be imposed like the one shown in the diagram at $16M. When such a constraint is imposed, we have **capital rationing** in that available capital dollars have to be rationed among projects.

Capital rationing creates a problem because projects are generally not divisible. In this case, we can't do part of project C, so we can't do it at all. That leaves some

Capital rationing involves selecting projects subject to a funding limitation.

Figure 10.6

Capital Rationing

unused money within the budget between the end of project B and the constraint. The rationing problem is to choose the best set of projects that fits into the capital constraint. By "best" we mean the set that maximizes NPV. In the illustration the choice appears to be easy because D, the next best project, just fits into the $3M space available. In some cases, however, the choice isn't so obvious. It's possible that the selection of projects that maximizes NPV could omit one of the higher-rated projects in favor of one or more lower-rated projects that fit better within the constraint.

Finding the best possible solution to a capital rationing problem involves using techniques from a field of mathematics known as constrained maximization. The subject is quite complex and beyond the scope of this book, but you should be aware of its existence.

Using sophisticated mathematics to find a precise solution to capital rationing problems implies attributing a great deal of accuracy to the NPV and IRR estimates of the projects being considered. In the next chapter we'll learn that such accuracy frequently isn't possible.

from the **CFO**

In practice, financial managers ration capital intuitively, choosing among projects for a variety of reasons, not all of which are strictly financial. In that way they make choices that are usually close to the best, but not exactly optimal.

QUESTIONS

1. Define "mutual exclusivity" and describe ways in which projects can be mutually exclusive.

2. Capital budgeting is based on the idea of identifying incremental cash flows, so overheads aren't generally included. Does this practice create a problem for a firm

which, over a long period of time, takes on a large number of projects that are just barely acceptable under capital budgeting rules?

3. Relate the idea of cost of capital to the opportunity cost concept (page 226). Is the cost of capital the opportunity cost of project money?

4. The payback technique is criticized for not using discounted cash flows. Under what conditions will this matter most? That is, under what patterns of cash flow will payback and NPV or IRR be likely to give different answers?

5. Explain the rationale behind the NPV method in your own words. Why is a higher NPV conceptually better than a lower one?

6. Projects A and B have approximately the same NPV. Their initial outlays are similar in size. Project A has early positive cash flows, and little or nothing is expected to come in later on. Project B has much larger positive cash flows than A, but they're further in the future. Can you make any general statement about which project might be better?

7. Suppose the present value of cash ins and outs is very close to balanced for a project to build a new $50 million factory, so that the NPV is +$25,000. The same company is thinking about buying a new trailer truck for $150,000. The NPV of projected cash flows associated with the truck is also about $25,000. Does this mean that the two projects are comparable? Is one more desirable than the other? If the cash flows have similar risks, are the projects equally risky? (*Hint:* Think in terms of the size of the investment placed at risk in relation to the financial rewards expected.)

8. Think about the cash flows associated with putting $100,000 in the bank for five years, assuming you draw out the interest each year and then close the account. Now think about a set of hypothetical cash flows associated with putting the same money in a business, operating for five years, and then selling out. Write an explanation of why the IRR on the business project is like the bank's interest rate. How are the investments different?

9. What is it about the cash flows associated with business projects that makes the NPV profile slope downward to the right? Would the NPV profile of any randomly selected set of positive and negative flows necessarily slope one way or the other? Why?

10. The following set of cash flows changes sign twice and has two IRR solutions. Identify the sign changes. Demonstrate mathematically that 25% and 400% are both solutions to the IRR equation.

C_0	C_1	C_2
$(320)	$2,000	$(2,000)

On the basis of this example, why would you expect multiple solutions to be an unusual problem in practice?

11. Under what conditions will the IRR and NPV methods give conflicting results for mutually exclusive decisions? Will they ever give conflicting results for stand-alone decisions? Why?

12. Why is the profitability index more appropriately described as a variation on the NPV technique than as a variation on the IRR technique?

13. Show that the profitability index (PI), the initial outlay (C_0), and the net present value (NPV) of a project are related by the following equation.

$$NPV = C_0(1 - PI)$$

(*Hint:* State both the NPV and the PI in terms of C_0 and the sum of all other cash flows.)

BUSINESS ANALYSIS

1. You are a financial analyst for the Ajax Company, which uses about $1 million of inventory per month. The purchasing manager has come to you for help with a buying decision. He can get a big discount on $15 million of inventory by buying it all at once. However, there is some risk of obsolescence when buying that far in advance. He understands that large purchases are frequently analyzed by means of capital budgeting techniques, and he asks for your help in deciding whether to buy the specially priced inventory. How would you advise him? Is capital budgeting appropriate?

2. Risk in capital projects is the probability that a project will earn less than expected. Make up and describe one hypothetical project in each of the replacement, expansion, and new venture categories. List a few ways that each might go wrong and cause the cash flows to be less favorable than expected. Can you think of situations in which projects could result in losses? Could the losses exceed the initial investment (C_0)?

3. Charlie Brown is thinking about starting Wing-It Airlines to fly a commuter route in and out of a major city. Four planes are on the market that will do the job, but each has different flight, load, and operating characteristics. Charlie is unsure of the demand for his service, and feels that it may depend to some extent on the type of plane chosen. Whether the business is feasible may depend on which airplane is used in conjunction with the demand estimate assumed. Are capital budgeting techniques appropriate for analysis of this problem? If so, is the issue a stand-alone or mutually exclusive decision?

4. The Budwell & Son Oil Company is looking at two drilling proposals. One project lasts for three years, costs $20 million to start, pays back quickly, and has an NPV of $15 million. The other project also costs about $20 million to start, but has an expected life of seven years, takes much longer to pay back, and has an NPV of $17 million. Mr. Budwell, the company's founder, favors the shorter project because of the quick investment recovery. His son Billy, however, has taken finance in college and insists that the only way to judge projects is on NPV. He therefore favors the longer project. They've engaged you as their financial advisor to settle the issue. How would you advise them?

5. Webley Corp. has a capital budget limited to $20 million. Five relatively high IRR projects are available that have initial investments totaling $15 million. They are all roughly the same size. A sixth project has an IRR only slightly lower than those of the first five but requires an $8 million investment. Several other smaller projects are available with IRRs quite a bit lower than the sixth. The president has stated that it's too bad the firm has to pass up the sixth project, but it just doesn't fit into the budget. How would you advise him?

PROBLEMS

1. Gander, Inc. is considering two projects with the following cash flows.

Year	Project X	Project Y
0	($100,000)	($100,000)
1	40,000	50,000
2	40,000	0
3	40,000	0
4	40,000	0
5	40,000	250,000

Gander uses the payback period method of capital budgeting and accepts only projects with payback periods of 3 years or less.

a. If the projects are presented as standalone opportunities which one(s) would Gander accept? If they were mutually exclusive and Gander disregarded its three year rule, which project would be chosen?

b. Is there a flaw in the thinking behind the correct answers to part a?

2. A project has the following cash flows.

C_0	C_1	C_2	C_3
$(700)	$200	$500	$244

a. What is the project's payback period?

b. Calculate the project's NPV at 12%.

c. Calculate the project's PI at 12%.

3. Calculate an IRR for the project in the previous problem using an iterative technique. (*Hint:* Start by guessing 15%.)

4. Clancy Inc. is considering a project with the following cash flows.

C_0	C_1	C_2	C_3
$(7,800)	$2,300	$3,500	$4,153

a. Clancy has a policy of rejecting all projects that don't pay back within three years and analyzing those that do more carefully with time value based methods. Does this project warrant further consideration?

b. Should Clancy accept the project based on its NPV if the company's cost of capital is 8%?

c. What conclusion will the firm reach based on PI?

5. Should the project being considered in the previous problem be accepted or rejected based on IRR? (*Hint:* Start by guessing 11% for IRR.)

6. Hamstring Inc. is considering a project with the following cash flows.

C_0	C_1	C_2	C_3	C_4
$(25,000)	$10,000	$12,000	$5,000	$8,000

The company is reluctant to consider projects with paybacks of more than three years. If projects pass the payback screen, they are considered further by means of the NPV and IRR methods. The firm's cost of capital is 9%.

a. What is the project's payback period? Should the project be considered further?
b. What is the project's NPV? Does NPV indicate acceptance on a stand-alone basis?
c. Calculate the project's IRR by using an iterative approach. Start by using the cost of capital and the NPV calculation from part (a). Does IRR indicate acceptance on a stand-alone basis?
d. What is the project's PI? Does PI indicate acceptance on a stand-alone basis?

7. Project Alpha requires an initial outlay of $35,000 and results in a single cash inflow of $56,367.50 after five years.

a. If the cost of capital is 8%, what are Alpha's NPV and PI? Is the project acceptable under each of these techniques?
b. What is project Alpha's IRR? Is it acceptable under IRR?
c. What are Alpha's NPV and PI if the cost of capital is 12%? Is the project acceptable under that condition?
d. What is Alpha's payback period? Does payback make much sense for a project like Alpha? Why or why not?

8. The Sampson Company is considering a project that requires an initial outlay of $75,000 and produces cash inflows of $20,806 each year for five years. Sampson's cost of capital is 10%.

a. Calculate the project's payback period by making a single division rather than accumulating cash inflows. Why is this possible in this case?
b. Calculate the project's IRR, recognizing the fact that the cash inflows are an annuity. Is the project acceptable? Did your calculation in this part result in any number(s) that were also calculated in part (a)? What is it about this problem that creates this similarity? Will this always happen in such cases?
c. What is the project's NPV? Is the project acceptable according to NPV rules?

9. Calculate the IRR, NPV, and PI for projects with the following cash flows. Do each NPV and PI calculation at costs of capital of 8% and 12%. Calculate IRRs to the nearest whole percent.

a. An initial outlay of $5,000 and inflows of $1,050 for seven years.
b. An initial outlay of $43,500 and inflows of $14,100 for four years.
c. An investment of $78,000 followed by 12 years of income of $11,500.
d. An outlay of $36,423 followed by receipts of $8,900 for six years.

10. Island Airlines Inc. needs to replace a short-haul commuter plane on one of its busier routes. Two aircraft are on the market that satisfy the general requirements of the route. One is more expensive than the other but has better fuel efficiency and load-bearing characteristics, which result in better long-term profitability. The useful life of both planes is expected to be about seven years, after which time both are assumed to have no value. Cash flow projections for the two aircraft follow.

	Low Cost	High Cost
Initial cost	$775,000	$950,000
Cash inflows, years 1 through 7	154,000	176,275

a. Calculate the payback period for each plane and select the best choice.
b. Calculate the IRR for each plane and select the best option. Use the fact that all the inflows can be represented by an annuity.
c. Compare the results of parts (a) and (b). Both should select the same option, but does one method result in a clearer choice than the other based on the relative sizes of the two payback periods versus the relative sizes of the two IRRs?
d. Calculate the NPV and PI of each project assuming a cost of capital of 6%. Use annuity methods. Which plane is selected by NPV? By PI?
e. Calculate the NPV and PI of each project, assuming the following costs of capital: 2%, 4%, 6%, 8%, and 10%. Use annuity methods. Is the same plane selected by NPV and PI at every level of cost of capital? Investigate the relative attractiveness of the two planes under each method.
f. Use the results of parts (b) and (e) to sketch the NPV profiles of the two proposed planes on the same set of axes. Show the IRRs on the graph. Would NPV and IRR ever give conflicting results? Why?

11. Calculate the NPV for the following projects.

a. An outflow of $7,000 followed by inflows of $3,000, $2,500, and $3,500 at one-year intervals at a cost of capital of 7%.
b. An initial outlay of $35,400 followed by inflows of $6,500 for three years and then a single inflow in the fourth year of $18,000 at a cost of capital of 9%. (Recognize the first three inflows as an annuity in your calculations.)
c. An initial outlay of $27,500 followed by an inflow of $3,000 followed by five years of inflows of $5,500 at a cost of capital of 10%. [Recognize the last five inflows as an annuity, but notice that it requires a treatment different from the annuity in part (b).]

12. Calculate the IRR for the following projects.

a. An initial outflow of $15,220 followed by inflows of $5,000, $6,000, and $6,500.
b. An initial outflow of $47,104 followed by inflows of $16,000, $17,000, and $18,000.

13. Calculate the NPV at 9% and the IRR for the following projects.

a. An initial outlay of $69,724 and an inflow of $15,000 followed by four consecutive inflows of $17,000.
b. An initial outlay of $25,424 followed by two zero cash years and then four years of inflows at $10,500.
c. An outlay of $10,672 followed by another outlay of $5,000 followed by five inflows of $5,000.

14. Calculate the NPV at 12% and the IRR for the following projects. Find IRRs to the nearest whole percent.

a. An initial outflow of $5,000 followed by three inflows of $2,000.
b. An initial outflow of $5,000 followed by inflows of $1,000, $2,000, and $3,000.
c. An initial outflow of $5,000 followed by inflows of $3,000, $2,000, and $1,000.
d. Notice that in parts (a), (b), and (c), a total of $6,000 is received over three years. Compare the NPVs and IRRs to see the impact of shifting $1,000 between years.

15. Grand Banks Mining Inc. plans a project to strip-mine a wilderness area. Setting up operations and initial digging will cost $5 million. The first year's operations

are expected to be slow and to net a positive cash flow of only $500,000. Then there will be four years of $2 million cash flows after which the ore will run out. Closing the mine and restoring the environment in the sixth year will cost $1 million.

a. Calculate the project's NPV at a cost of capital of 12%.
b. Calculate the project's IRR to the nearest whole percent.

16. Bagel Pantry Inc. is considering two mutually exclusive projects with widely differing lives. The company's cost of capital is 12%. The project cash flows are summarized as follows.

	Project A	Project B
C_0	$(25,000)	$(23,000)
C_1	14,742	6,641
C_2	14,742	6,641
C_3	14,742	6,641
C_4		6,641
C_5		6,641
C_6		6,641
C_7		6,641
C_8		6,641
C_9		6,641

a. Compare the projects using payback.
b. Compare the projects using NPV.
c. Compare the projects using IRR.
d. Compare the projects using the replacement chain approach.
e. Compare the projects using the EAA method.
f. Choose a project and justify your choice.

CALCULATOR PROBLEMS

The problems in this section should be solved using a financial calculator. See pages 436–437.

17. Callaway Associates, Inc. is considering the following mutually exclusive projects. Callaway's Cost of capital is 12%.

Year	Project A	Project B
0	($80,000)	($80,000)
1	$44,000	$65,000
2	$34,000	$30,000
3	$14,000	$ 0
4	$14,000	$ 5,000

a. Calculate each project's NPV and IRR.
b. Which project should be undertaken? Why?

18. Tutak Industries is considering a project requiring an initial investment of $200,000 followed by annual cash inflows of $45,000 for the next six years. A second six-year project has an initial outlay of $325,000.

   a. How much would the second project have to generate in annual cash flows to have the same IRR as the first?
   b. If Tutak's cost of capital is 8%, how much would the second project have to generate in annual cash flows to have the same NPV as the first project?

19. Provide the missing information for the following projects.

Project	Initial Investment	Length (in years)	Annual Cash Flow	Cost of Capital	NPV
A	$100,000	5	$35,000	8%	?
B	200,000	4	?	13	$35,000
C	300,000	7	50,000	?	15,000
D	400,000	?	56,098	9	20,000
E	?	6	75,000	10	25,000

20. Calculate IRRs for the projects in the previous problem.

21. Huron Valley Homes is considering a project requiring a $1 million initial investment. Expected cash inflows will be $25,000 in the first year, $100,000 in the second year, and $200,000 per year for the next six years.

   a. Calculate the project's IRR and the NPV assuming an 8% cost of capital.
   b. How much would each of the last six payments have to be to make the project's NPV $100,000?

22. Consider two mutually exclusive projects, A and B. Project A requires an initial cash outlay of $100,000 followed by five years of $30,000 cash inflows. Project B requires an initial cash outlay of $240,000 with cash inflows of $40,000 in the first two years, $80,000 in the next two years, and $100,000 in the fifth year.

   a. Compute the IRR for each project.
   b. Compute the NPV for each project for each of the following costs of capital: 0%, 4%, 8%, 12%, and 16%, and record your results in a table.
   c. For which costs of capital do the IRR and NPV methods select the same project?
   d. Examine the table created in part (b) and determine the costs of capital between which the methods begin to select different projects. Is your answer consistent with the result of part (a)? Explain your answer in terms of NPV profiles.

23. Kneelson and Botes Inc. (K&B) is a construction company that does road and bridge work for the state highway authority. The state government solicits bids on construction projects from private contractors. The winning contractor is chosen based on its bid price as well as its perceived ability to do the work.

   Sophisticated contractors develop bids using capital budgeting techniques because most projects require cash outlays for hiring, equipment, and materials before getting started (C_0). After that the state makes progress payments to cover costs and profits until the job is finished ($C_1 \ldots C_n$).

   Contractors know that even after they've won a bid, realizing the planned profits and cash flows isn't assured in part because government budgets can change while construction progresses. If funding is up, officials tend to add to the work

originally ordered leading to increased profits and cash flows. But if funding is down, officials start to nit pick the contract looking for cost savings, which generally leads to lower cash inflows. State budget projections are fairly good for a year or two, but tend to be inaccurate over longer periods.

K&B has been offered two, four-year contracts, but doesn't have enough cash or management depth to take on both (mutually exclusive because of resource limitations). One project involves road repair, most of which will be done and paid quickly. The other requires working on a new bridge. The bulk of the cash inflows on bridge projects generally occur near completion.

K&B's estimating department has put together the following projections of the two projects' cash flows:

	($ooo)	
	Road Repair	**Bridge Work**
C_0	($3,000)	($4,500)
C_1	3,000	100
C_2	2,000	2,000
C_3	1,000	3,000
C_4	100	4,500

K&B doesn't know its exact cost of capital, but feels it's between 10 and 15%. This is not uncommon in smaller companies. (In Chapter 13 we'll learn that estimating the cost of capital can be difficult and less than precise for firms of any size.)

The company has hired you as a financial consultant to make a recommendation as to which project to accept.

a. Calculate the payback period for both projects. Which does payback choose?
b. Calculate the IRR for both projects. Which does the IRR method choose? Is the choice clear or is it a close decision? Is the choice consistent with the result of the payback method?
c. Calculate NPVs for both projects for costs of capital from 10 to 15% in 1% increments. Then plot both projects' NPV profiles on a graph similar to that shown in Figure 10.2 on page 435. Does the NPV method give a meaningful result? If so is it consistent with the results of the payback and IRR methods? Which method is theoretically the best? Does that help in this situation?
d. You must make a recommendation to K&B's management regardless of any technical difficulties you've encountered. Provide another, less quantitative argument that tends to support one project over the other. (*Hint*: See question 6 on page 446 and Business Analysis 4 on page 447.)
e. What is your recommendation and why?

24. Haley Motors is considering a maintenance contract for its heavy equipment. One firm has offered Haley a four-year contract for $100,000. Another firm has offered an eight-year contract for $165,000. Haley will be able to save $34,000 per year under either contract because its employees will no longer have to do the work themselves.

a. If Haley's cost of capital is 10%, which project should be selected? Use both the replacement chain and the equivalent annual annuity (EAA) method to justify your answer.

b. If Haley's cost of capital is 12%, does it change the decision? What about 14%?

25. Cassidy and Sons is reviewing a project with an initial cash outflow of $250,000. An additional $100,000 will have to be invested after the first year, followed by an additional investment of $50,000 at the end of the second year. Beginning at the end of year 3, the project is expected to generate cash flows of $90,000 per year for the next eight years.

   a. Calculate the project's payback period, IRR, and its NPV and PI at a cost of capital of 8%.
   b. What concerns might Cassidy have regarding this project beyond the financial calculations from part (a)?

26. Zuker Distributors handles the warehousing of perishable foods and is considering replacing one of its primary cold storage units. One supplier has offered a unit for $250,000 with an expected life of 10 years. The unit is projected to reduce electricity costs by $50,000 per year. However, it requires a $20,000 refurbishing every two years, beginning two years after purchase. Another supplier has offered a cold storage unit with similar capabilities for $300,000. It will produce the same savings in electricity costs, but requires refurbishing every five years at a cost of $40,000. Zuker's cost of capital is 8.5%. Use NPV to determine which cold storage unit Zuker should select.

27. Griffin-Kornberg is reviewing the following projects for next year's capital program.

Project	Initial Investment	Length in Years	Annual Cash Flow
A	$3.0 million	6	$ 719,374
B	3.5 million	5	970,934
C	4.0 million	7	904,443
D	5.0 million	4	1,716,024
E	6.0 million	6	1,500,919
F	7.0 million	5	1,941,868
G	8.0 million	7	1,725,240

Projects A and B are mutually exclusive and so are Projects D and E. Griffin-Kornberg has a 9% cost of capital and a maximum of $14 million to spend on capital projects next year. Use capital rationing to determine which projects should be included in Griffin-Kornberg's capital program.

INTERNET PROBLEM

28. For a brief discussion of the rationale behind capital budgeting, go to http://www.netmba.com/finance/capital/budgeting/ and read about the relationship between the process and pleasing shareholders. Write a summary of the reason(s) why the author emphasizes NPV over other methods of analysis.

COMPUTER PROBLEMS

Developing Software

29. Write a spreadsheet program to calculate the NPV of a project with an irregular pattern of cash flows for up to 10 periods without using the spreadsheet software's NPV function. Essentially, the task is to program equation 10.1 with n = 10.

    First input the interest rate (k) in a single cell.

    Next set up three horizontal rows of 11 cells (including C_0). The top row will receive the cash flows as inputs.

    Program the present value factor for each period into the second row of cells using the interest rate you input earlier as follows.

Period	0	1	2	. . .	. . .	10
Factor	1	$\dfrac{1}{1+k}$	$\dfrac{1}{(1+k)^2}$	. . .	. . .	$\dfrac{1}{(1+k)^{10}}$

Note that we're calling the interest rate k, but it will appear as a cell name in your program.

Next form the third row by multiplying the top two cells in each column together. This makes the third row the present value of each cash flow.

Finally, sum the values along the third row in another cell to form the project's NPV.

Notice that your program will handle a project of less than 10 periods if you simply input zero (or leave blank) the cash flow cells from n+1 to 10.

Also notice that you can easily extend your program to any reasonable number of periods by extending the horizontal rows and the programming logic. Test your program on the data in Example 10.3 on pages 428–430 to make sure it works correctly.

30. The Tallahassee Motor Company is thinking of automating one of its production facilities. The equipment required will cost a total of $10 million and is expected to last 10 years. The company's cost of capital is 9%. The project's benefits include labor savings and a quality improvement that will lower warranty costs. Savings are estimated as follows.

Year	Cost Savings ($ooo)
1	$ 574
2	864
3	1,246
4	2,748
5	3,367
6	2,437
7	2,276
8	1,839
9	1,264
10	623

a. Use the program developed in the previous problem to find the project's NPV. Is the project acceptable?

b. Use the program to develop the data for an NPV profile. Evaluate the NPV for interest rates (costs of capital) from 6% to 14%.

c. Use the program to iteratively find the project's IRR to one hundredth of a percent.